Eliminating transplant waiting time inequities – With an application to kidney allocation in the USA

Joris van de Klundert, Liana van der Hagen, Aniek Markus

# Abstract

This research models, analyzes and optimizes equity of transplant waiting times and probabilities using queuing models, network ﬂows, and Rawls’ Theory of Justice, addressing inequities resulting from blood type incompatibilities, which are inter-related to ethnic diﬀerences in patient and donor rates.

# Introduction

Organ transplant wait lists can be modeled as single server queues with patient arrival rate and organ arrival rate . Unfortunately, organ transplant wait lists often have a utilization rate of . The queue length nevertheless stabilizes because of patients leaving the queue without receiving a DTx (deceased donor transplantation). Such behaviors have been modeled as abandonment or reneging in the queuing literature. The persistent relative scarcity of organ supply has brought along challenging allocation problems and inequalities in waiting times and transplant probabilities.

Our research aim is to model existing policies and minimize the resulting ethnic inequities in DTx waiting times and probabilities. We model inequity following Rawls’ Theory of Justice and ﬁrst turn to addressing underlying blood type related inequities. The models disregard various patient characteristics which are important in practice for organ allocation at the patient level.

# Modelling and theoretical background

## Allocation, blood types and ethnicity

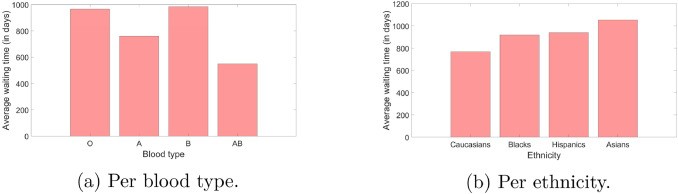
Policies:

* *identical allocation* aims to avoid inequity by only allowing transplantations from donors to patients of the same type,
* *compatible allocation* allows transplantations to patients with blood types that are compatible with the blood type of a donor.
* *KAS*  (kidney allocation system) allows for identical allocation and for allocation of A organs to AB patients.

Immagine che contiene testo, orologio, clipart

Descrizione generata automaticamente

Diﬀerent policies may allocate organs diﬀerently and thus impact transplant waiting times and probabilities per blood type diﬀerently. The diﬀerences in disease prevalence and donation rates among ethnic groups importantly cause patient and organ arrival rate diﬀerences per blood type and hence per ethnic group.



Using a network ﬂow formulation we present necessary and suﬃcient conditions for equal waiting times and transplant probabilities for patients from diﬀerent blood types in case of Poisson patient and organ arrivals. We subsequently develop an algorithm to minimize inequity when these conditions are violated. These models and methods are extended to address ethnic inequality and inequity.

## Equity in transplant waiting times

Health inequality is an observable health diﬀerence between subgroups within a population. Health equity refers to the absence of health inequalities. Waiting time inequalities between sub populations may arise as a result of diﬀerences in the relative volumes of organs allocated.

Maximal allocation is achieved when all available organs are allocated. If maximality and equality of waiting times cannot be simultaneously achieved, then inequalities are considered unavoidable and necessary. Within the set of maximal allocations, we pursue equity by avoiding other inequalities.

We further formalize (in)equity based on Rawls’ Theory of Justice: *each person is to have an equal right to the most extensive basic liberty compatible with a similar liberty for others*.

* Inequalities that result from improving waiting time for some sub populations are fair and just if they recursively minimize the maximum waiting times for the remaining patient sub populations.
* Diﬀerences in transplant probabilities are fair and just if they recursively maximize minimum transplant probabilities.

Hence, in order to minimize inequity, we set out to ﬁnd maximal allocations that recursively minimize the maximum waiting time and maximize minimum transplant probability.

## Queuing theory foundations of waiting times

queuing models have formed the most common formal approach to optimizing allocation of organs to patients for deceased donation. For a patient receiving a transplant, the actual time until transplant is the waiting time plus the service time, i.e. the sojourn time. In practice, the service time is often much smaller (hours or days) than the preceding time in the queue (years). Hence the waiting time until service is commonly considered for minimization.

The expected number of newly arriving patients per time period is denoted by , while denotes the expected number of donor organs per time period available to service the patients in the queue. The utilization rate is yet the queue length does not grow indeﬁnitely because patients renege. The renege probability per time unit () is assumed to be constant over time and independent of queue length. Hence, the time until reneging follows an exponential distribution.

Let be the length of the wait list in time period *t*. Under the realistic assumptions that the queue is never empty and that patient arrivals continue to exceed donor organ arrivals (), then transitions from time periods by: 🡪 .

A stable equilibrium can still be obtained for the queue length due to the reneging. In this equilibrium, per time period,

* patients enter the waiting list,
* patients receive a DTx,
* patients renege.

The equilibrium time in the queue is: 🡪 .

As the queue length approaches inﬁnity:

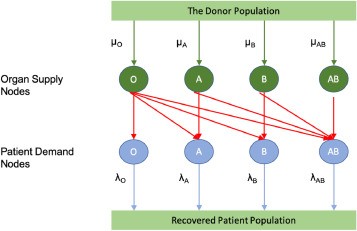
* the asymptotic stationary expected waiting time (*wait time*) converges to 🡪 ;
* the asymptotic stationary expected transplant probabilities (*Tx probability*)

converge to 🡪 .

Our objectives are to (recursively) minimize the maximum wait time and maximize the minimum Tx probability.

## Donor allocation as network ﬂow

DTx allocation policies determine how many organs of each type to allocate to patients of each compatible type. The total number of organs allocated to each type then forms the organ arrival rates for the corresponding patients. Together with the patient arrival rates, these allocated organ arrival rates determine the wait times and Tx probabilities.



* : donor organs of type *x* that becomes available.

For each blood type , the capacity of the arc is set at , which is the number of donor organs of type *x* available per time unit.

For any feasible ﬂow *f* in *G*, let represent the corresponding ﬂow on arc .

* : link from the organ *x* supply to a demand of a compatible type *y*.

It has inﬁnite capacity, for any organ that becomes available can be utilized for transplantation.

Because of ﬂow conservation it must hold that 🡪 , with the ﬂow on .

Maximal allocations satisfy .

* : all organs allocated to patients of type *y* per time period.

Flow conservation implies that 🡪 .

For any ﬂow *f*  and for all , the represent Poisson organ arrivals to patients of type *y* when all the organs of type *x* are randomly allocated over the patients type compatible with *x* with probability .

The values of are capacity limits for , as we cannot transplant more organs to type than the number of patients arriving.

The allocation problem then becomes to ﬁnd a maximal feasible ﬂow *f* in *G* that results in most equitable wait times and Tx probabilities.

# Equity and blood types

The Max-Flow Min-Cut theorem speciﬁes that the value of a maximum ﬂow in *G* is equal to the value of a minimum cut in *G*. A cut is deﬁned as a partition of the vertices of *G* into two sets:

* *S* which contains the source,
* *T* which contains the sink.

The value of a cut equals the sum of the capacities of the arcs included in the cut-set.

As the red arcs have inﬁnite capacity, they will not be included in any minimum cut set. A minimum cut is characterized by the blood types in the sets and that minimizing . Here, implicates that is in the cut-set and means that is in the cut-set.

For any feasible ﬂow *f* in *G* and for each blood type *x*, implies a wait time for the patients receiving a DTx of: 🡪 .

*f* yields equal wait times if there exists a rational number such that .

The maximal allocation translates to ﬁnding maximal ﬂows *f* satisfying .

Deﬁne to be identical to *G* except for adjusting the capacities of the blue arcs to , so that the total capacity of the blue arcs equals the total capacity of the green arcs 🡪 .

**Theorem 1**

*A maximal allocation of donor organs to patients resulting in equal wait times exists if and only if a feasible ﬂow f of value exists in .*

Blood type identical allocation yields equal and equitable wait times among blood types in case donor organ arrival rates and patient arrival rates are proportional to the blood type distributions in a population. Hence, the complexity of ﬁnding equitable allocations among sub populations of diﬀerent blood types arises from disproportional arrival rates.

In case a ﬂow of value exists in , both the set of green arcs and the set of blue arcs form a cut-set.

**Theorem 2**

*If and only if for every minimum cut in it holds that and , then*

* *,*
* *,*
* *the maximum ﬂow has value less than .*

If the maximum ﬂow *f*  has value strictly less than , the cut partitions into two components:

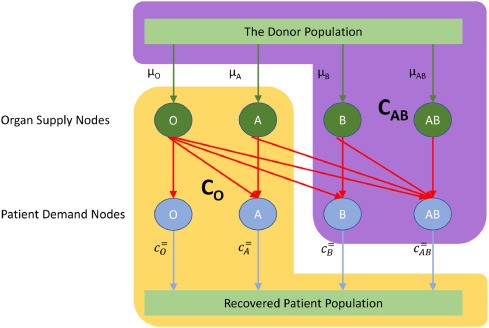
* one contains the source, connected to all supply nodes of types and subsequently via the red arcs to compatible demand nodes. It contains the supply and demand nodes of *AB* and will therefore be referred to as .
* the other containing the sink, symmetrically consists of the demand nodes of type , together with the supply nodes of blood types compatible with these demand nodes. It necessarily contains the demand and supply nodes for type *O* and will therefore be referred to as .

We deﬁne a cut by blood types, rather than by vertices. This means that and for minimum cut .

Because is a minimum cut, it must hold that

🡪

🡪



Considering the case for which the value of the minimum cut is strictly less than , at least one of these inequalities is strict and hence at least one of the types in must have longer wait times than at least one of the types in for any maximum allocation.

Even though maximality and equality may not be jointly attainable, one can still pursue equity for maximal allocations following Rawls’ recursive minimax principles. Develop a recursive algorithm to obtain an equitable solution corresponding to these principles.

Let:

* be the subgraph of *G* induced by the types in ,
* be equal to , except for adjusting the capacities of the blue arcs to ,
* and deﬁne the cut-set corresponding to a minimum cut in .

**Theorem 3**

*If for every minimum cut in () it holds that () and (), then the type 0 (AB) demand and supply nodes are in the component which also contains the sink (source) and the maximum ﬂow has value less than ().*

A minimum cut partitions it into two components, one of which contains the demand and supply nodes for blood type *O (AB)*.

The results are combined in Algorithm 1 in recursive function EquitableFLow() to obtain an equitable ﬂow according to Rawls’ Theory of Justice.

Immagine che contiene testo

Descrizione generata automaticamente

The algorithm is initiated with a ﬁrst call to EquitableFlow(*G=*,*X*). The output is an equitable ﬂow vector .

# Equity and ethnicity

As blood type prevalences vary over ethnic groups, equity among blood types is no guarantee for equity among ethnic groups. The next step is therefore to address ethnic inequities, which persist in practice.

Let be an example of the set of all ethnic groups. Let be the arrival rate of patients of blood type *x* and ethnicity *v* and let be the corresponding organ arrival rate. Then, by deﬁnition and .

Immagine che contiene tavolo

Descrizione generata automaticamente

Immagine che contiene tavolo

Descrizione generata automaticamente

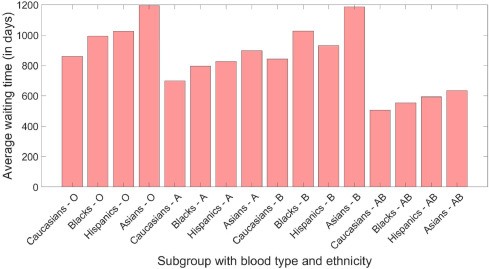
As Caucasians are relatively more prevalent among donors, identical allocation would result in a higher utilization rate for blood type *A* than for blood type *O*.

The question rises whether these inequalities can be reduced to become as equitable as possible. In the remainder, we disregard the ethnic origin of the organs () and only consider . The analysis therefore focuses on the .

**Theorem 5**

*Let f be an equitable allocation as determined by Equitable Flow. Then, is an equitable allocation for the case with blood type and ethnicity.*

As consequence, for the case in which ethnic equity is explicitly considered together with blood type equity, equity can be maximized by ﬁrstly ﬁnding an allocation that maximizes equity among the blood types and subsequently allocating organs per blood type proportionally to demand of the ethnic groups for that blood type. This can be achieved by merging all patients into one queue per blood type and subsequent implementation of any policy for which allocation probabilities are subsequently independent of ethnicity.



While it may be possible to reduce ethnic inequalities, this necessarily implies inequity according to the Theory of Justice.

# Equity of deceased donor kidney allocation in the United States

Considerable differences exist between organ and patient arrival rates per ethnic group and subsequently per blood type, especially for blood types *A* and *B*.

Over the years 2014–2017:

* 30.816 new patients entered the DTx wait list on average per year,
* 12.862 patients received a DTx on average per year.

The queue length:

* started at 96.848 on January 1, 2014,
* peaked at 99.172 on December 31, 2014,
* decreased to 92.685 by the end of 2017.

Calculated over the average of queue lengths, the annual renege rate has been 0.196 (ap- proximately one in five patients leaves the queue annually without transplant).

Disregarding blood type incompatibilities for the time being, these total numbers would yield:

* an equilibrium wait list length of 🡪
* a corresponding wait time of 🡪 years (1085 days).
* a Tx probability of 🡪 .

Immagine che contiene tavolo

Descrizione generata automaticamente

For identical allocation, type *B* patients can expect to wait 287 days longer than type *A* patients. The differences among ethnic groups are much smaller, with a maximum of 51 days between Caucasians and Asian Americans. Given that compatible allocation is feasible and yields comparable patient outcomes, the inequalities resulting from identical allocation can be viewed to be avoidable and hence inequitable, in so far as they exceed the inequalities resulting from compatible allocation.

The wait times and Tx probabilities presented for blood type compatible allocation are obtained by Algorithm Equitable Flow. The Algorithm separates component from component . The corresponding equitable wait times are equal for types O and B (through allocating some type O organs to type B patients) and for types A and AB (through allocating some type A organs to type AB patients). The resulting wait time differences are much smaller than for the case of identical allocation, yet still considerable. Type O and B patients wait 144 days longer than type A and AB patients. Translated to ethnic groups however, the equitable wait time inequalities are at most 18 days and the Tx probabilities differ by 0.013 at most.

The inequalities of compatible allocation can still be regarded as avoidable (and hence inequitable) as they do not utilize type A2 to B and A2B to B transplants, as practiced by UNOS since December 2014. The sub type A2 (A2B) makes up around 20% of the type A (AB) population. The formal analysis of the KAS policy which includes such transplants requires to modify the compatibility graph. We modify it by adding donation by type A2 and A2B donors to type B patients. As type AB has a high utilization rate and low prevalence, we disregard A2B to B donation in the remainder (without loss of optimality). In the presented results, we applied the Algorithm allowing for allocations from type A to B, rather than the more restricted KAS which only allows for donations from type A2 to B.

Around 9,8% of type A donor organs are allocated to type B patients. The only other non-identical allocations are from A to AB. The types A, B and AB have equal wait times without requiring A2B to B allocations. The equitable solution obtained when allowing for this additional allocation possibility has near perfect equality among blood types and ethnic groups. The total expected wait list length is 91.565.

# Discussion and conclusions

Allocations maximizing blood type equity can be straightforwardly translated to solutions jointly maximizing blood type and ethnic equity. Waiting time inequalities among ethnic groups cannot be further reduced without increasing inequalities among blood type. The inequalities arise from diﬀerences in organ and patient arrival rates among ethnic groups.

The results for blood type identical allocation yield considerable inequalities among blood types and rather modest ethnic inequalities. The results for compatible allocations yield quite limited inequalities for both. Inequalities, and hence inequities, virtually disappear when additionally allowing for *A2* to *B* transplantation as currently practiced in KAS.

There are other factors inﬂuencing the waiting times that are not included in the presented allocation model. Firstly, HLA proﬁles and HLA antibody proﬁles vary signiﬁcantly among ethnic groups, creating subsequent diﬀerences in waiting time and transplant probabilities. Secondly, health state and time on dialysis at time of enrolment vary among ethnic groups, which may subsequently lead to diﬀerences in priority and renege rates.